independently by Morris[3] who derived the stiffness matrix relative to an orthogonal coordinate system common to both ends of the member. For a study of curved-girder bridges, Sawko[4] derived the flexibility matrix for a curved beam element, but did not give an explicit form for the stiffness matrix. Numerical studies were discussed in all three papers.

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# Comments on "Hypersonic Flow Past an Unyawed Cone" and "Shoulder Pressures for Slender Cone-Afterbody Combinations in Hypersonic Flow"

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In Ref. 1, Rasmussen has developed an interesting procedure for calculating the hypersonic flow past an unyawed cone. In this procedure, the stream function differential equation in the hypersonic small-disturbance limit is recast as an integral equation, which is then solved by successive approximations. The approximate solution resulting from this analysis has subsequently been used in Ref. 2 to calculate the shoulder pressure on a cone-afterbody configuration.

In Ref. 1, the shock angle is computed from the zeroth-order approximation for the stream function. The surface pressure, on the other hand, is derived from the first-order approximation for the stream function but, in the process, the zeroth-order solution for the shock angle is incorporated. Thus, it seems that a certain degree of arbitrariness is introduced. Nevertheless, the proposed formulas have the advantage of being simple as well as accurate (provided that the hypersonic similarity parameter $K$ is not too small). Higher-order approximations presumably would improve the accuracy of the solutions for small values of $K$ (when the shock does not lie so close to the body) but such calculations would run into severe algebraic complications.

Now, it is interesting to observe that the proposed formulas for shock angle and surface pressure actually are constant-density solutions. In fact, the hypersonic small-disturbance equations for plane or axi-symmetric flow admit of constant-density solutions in closed explicit form for arbitrary body shape. The simplified equations are

$$(\partial/\partial y)[vy^j] = 0, \ \partial v/\partial x + v\partial v/\partial y = -\tfrac{1}{2}\epsilon\partial C_p/\partial y$$

with shock conditions $v_s = (1-\epsilon)G'$, $C_{p_s} = 2(1-\epsilon)G'^2$ and surface tangency condition $v_b = F'$. Conventional notation has been used. $F(x)$ and $G(x)$ represent the body shape and shock shape; $\epsilon$ is the preshock-postshock density ratio; $v$ is nondimensionalized by $u_\infty$. The foregoing system is readily solved, the results being

$$v = \begin{cases} F' & (j=0) \\ FF'/y & (j=1) \end{cases}$$

$$G = \begin{cases} F/(1-\epsilon) & (j=0) \\ F/(1-\epsilon)^{1/2} & (j=1) \end{cases}$$

$$C_p = \begin{cases} [2/(1-\epsilon)](F'^2 + FF'') - (2/\epsilon)(y-F)F'' & (j=0) \\ \dfrac{1}{\epsilon}(F'^2 + FF'')\ln\left[\dfrac{F^2}{(1-\epsilon)y^2}\right] + F'^2 + \dfrac{1}{\epsilon} \times (1 - F^2/y^2)F'^2 & (j=1) \end{cases}$$

For conical flow, we find,

$$\frac{C_p}{\theta_b^2} = 1 + \frac{1}{\epsilon}\left[\ln\left(\frac{\theta_b}{\theta}\right) - \left(\frac{\theta_b}{\theta}\right)^2 + \ln(1-\epsilon)^{-1} + 1\right]$$

where we have replaced $y$ by $\theta x$.

Specializing to a perfect gas with constant specific heats and adiabatic index $\gamma$, we further find,

$$\epsilon = [(\gamma-1)K^2 + 2]/[(\gamma+1)K^2 + 2]$$

where $K = M_\infty\theta_b$. Substitution into the solutions for shock angle and pressure coefficient yields results identical with those obtained by Rasmussen's procedure.

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Received November 5, 1969; revision received March 13, 1970.

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